Mathematical and Statistical Techniques

F.Y.B.Com. Semester- I

Unit 1 Shares and Mutual Funds

i) Shares is the smallest unit of the capital of a company.

Buying: Actual price of the share = Market value + Brokerage per share

Selling: Actual price of the share = Market value - Brokerage per share

Number of shares purchased / sold = $\frac{Total\ amount}{Price\ per\ share}$

Dividend is a part of the profit made by the company is distributed amongst the shareholders.

Dividend per share = Face value x rate of dividend

Total Dividend = Face value x rate of dividend x number of shares

Bonus shares: profit made by the company is distributed amongst the shareholders in the form of shares.

ii)Mutual Fund is a pool of money collected from investors and invest in stocks.

Net Asset value (N.A.V) = $\frac{Current\ value\ all\ assets - Liabilities}{Total\ number\ of\ units}$

Entry load and Exit load are the brokerage while buying and selling the mutual fund respectively.

Buying: Actual price of the Mutual fund = N.A.V + Entry load per share

Selling: Actual price of the Mutual fund = N.A.V - Exit load per share.

Systematic Investment plan(S.I.P)

Qn. An investor joined the S.I.P scheme, for a mutual fund, under which he would invest Rs. 750 for 4 months. If the N.A.Vs for each month are Rs.75,Rs.60,Rs.25 and Rs.50, find the average unit cost using Rupee Cost Averaging method and compare it with Arithmetic Mean of prices if the entry load was 2% through out, correct to 4 decimal places.

Ans:

Months	Sum invested	NAV	Entry load 2%	Price per unit	Number of units.
1	750	75	1.5	76.5	9.8039
2	750	60	1.2	61.2	12.2549
3	750	25	0.5	25.5	29.4118
4	750	50	1	51	14.7059
Total	3000			214.2	66.1765

Rupee cost average = $\frac{Total\ amount\ invested}{Total\ number\ of\ units}$ = $\frac{3000}{66.1765}$ = 45.3333

A.M. of prices = $\frac{Sum\ of\ all\ prices}{Number\ of\ months}$ = $\frac{214.2}{4}$ = 53.55

Thus the average price using Rupee cost method is less than A.M of prices.

Unit 2 Permutation, Combination and L.P.P

Factorial Notation:

n ! = n (n-1) (n-2) (n-3)………3.2.1

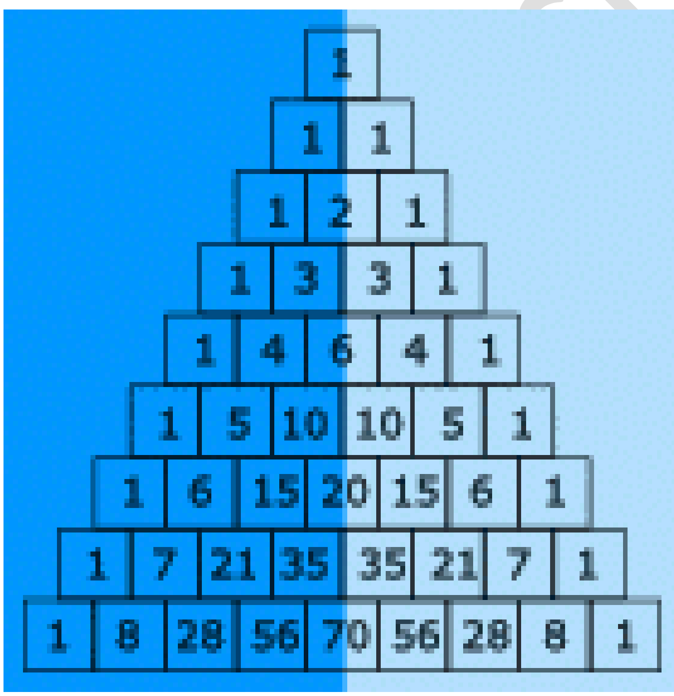
Permutation is an **arrangement** of r things out of n things.

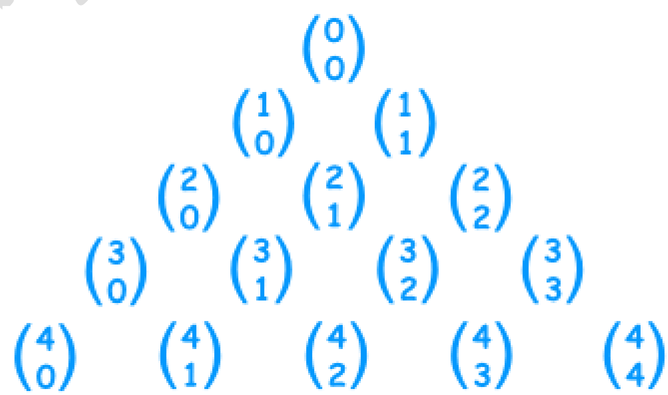
$$P(n,r) = {}^{n}P_{r} = {}_{n}P_{r} = \frac{n!}{(n-r)!}$$

Combination is a **selection** of r things out of n things

$$C(n,r) = {}^{n}C_{r} = {}_{n}C_{r} = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Pascal's Triangle





Linear Programming Problems (L.P.P)

Example 1 Solve the following linear programming problem graphically:

Maximise $Z = 4x + y$ ... (1)

subject to the constraints:

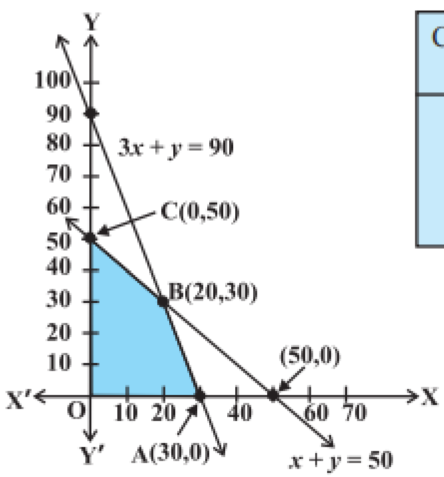
$$x + y \leq 50 \quad \text{... (2)}$$

$$3x + y \leq 90 \quad \text{... (3)}$$

$$x \geq 0, y \geq 0 \quad \text{... (4)}$$

Solution The shaded region in Fig 12.2 is the feasible region determined by the system of constraints (2) to (4). We observe that the feasible region OABC is **bounded.** So, we now use Corner Point Method to determine the maximum value of Z.

The coordinates of the corner points O, A, B and C are (0, 0), (30, 0), (20, 30) and (0, 50) respectively. Now we evaluate Z at each corner point.



Corner Point	Corresponding value of Z	
(0, 0)	0	
(30, 0)	**120** ←	Maximum
(20, 30)	110	
(0, 50)	50	

Fig 12.2

Hence, maximum value of Z is 120 at the point (30, 0).

Example 2 Solve the following linear programming problem graphically:

Minimise $Z = 200x + 500y$ ... (1)

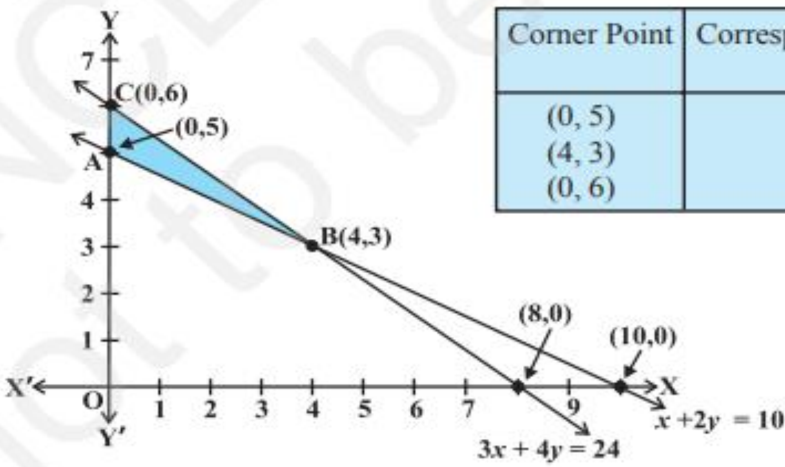
subject to the constraints:

$$x + 2y \geq 10 \quad \text{... (2)}$$

$$3x + 4y \leq 24 \quad \text{... (3)}$$

$$x \geq 0, y \geq 0 \quad \text{... (4)}$$

Solution The shaded region in Fig 12.3 is the feasible region ABC determined by the system of constraints (2) to (4), which is **bounded**. The coordinates of corner points



Corner Point	Corresponding value of Z	
(0, 5)	2500	
(4, 3)	**2300** ←	Minimum
(0, 6)	3000	

A, B and C are (0,5), (4,3) and (0,6) respectively. Now we evaluate $Z = 200x + 500y$ at these points.

Hence, minimum value of Z is 2300 attained at the point (4, 3)

Example 3 (Diet problem): A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food 'II' contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 50 per kg to purchase Food 'I' and Rs 70 per kg to purchase Food 'II'. Formulate this problem as a linear programming problem to minimise the cost of such a mixture.

Solution Let the mixture contain x kg of Food 'I' and y kg of Food 'II'. Clearly, $x \geq 0$, $y \geq 0$. We make the following table from the given data:

Resources	Food		Requirement
	I (x)	II (y)	
Vitamin A (units/kg)	2	1	8
Vitamin C (units/kg)	1	2	10
Cost (Rs/kg)	50	70	

Since the mixture must contain at least 8 units of vitamin A and 10 units of vitamin C, we have the constraints:

$$2x + y \geq 8$$

$$x + 2y \geq 10$$

Total cost Z of purchasing x kg of food 'I' and y kg of Food 'II' is

$$Z = 50x + 70y$$

Hence, the mathematical formulation of the problem is:

Minimise $\quad Z = 50x + 70y \quad$ ... (1)

subject to the constraints:

$$2x + y \geq 8 \quad \text{... (2)}$$

$$x + 2y \geq 10 \quad \text{... (3)}$$

$$x, y \geq 0 \quad \text{... (4)}$$

Example 4 (Manufacturing problem) A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week?

Solution Suppose x is the number of pieces of Model A and y is the number of pieces of Model B. Then

$$\text{Total profit (in Rs)} = 8000\,x + 12000\,y$$

Let $$Z = 8000\,x + 12000\,y$$

We now have the following mathematical model for the given problem.

Maximise $Z = 8000\,x + 12000\,y$ ... (1)

subject to the constraints:

$9x + 12y \le 180$ (Fabricating constraint)

i.e. $3x + 4y \le 60$ ... (2)

$x + 3y \le 30$ (Finishing constraint) ... (3)

$x \ge 0, y \ge 0$ (non-negative constraint) ... (4)

Measures of Central Tendency

1)Mean

$\bar{x} = \frac{\sum x}{n}$ (Ungrouped data), $\bar{x} = \frac{\sum fx}{\sum f} = \frac{\sum fx}{N}$ (Grouped data)

Combined mean ($\bar{x}$) = $\frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1 + n_2 + n_3}$

Weighted Arithmetic mean ($\bar{x}_w$) = $\frac{\sum wx}{\sum w}$

2)Mode = $l_1 + \frac{(l_2 - l_1)(f_1 - f_0)}{2f_1 - f_2 - f_0}$ or $l_1 + \frac{(l_2 - l_1)(f_1 - f_0)}{(f_1 - f_0) + (f_1 - f_2)}$

3)Median = $l_1 + \frac{(l_2 - l_1)(\frac{N}{2} - cf)}{f}$

4)Quartiles

$Q_1 = l_1 + \frac{(l_2 - l_1)(\frac{N}{4} - cf)}{f}$, $Q_2\ (Median) = l_1 + \frac{(l_2 - l_1)(\frac{N}{2} - cf)}{f}$,

$Q_3 = l_1 + \frac{(l_2 - l_1)(\frac{3N}{4} - cf)}{f}$

5)Deciles $(D_k) = l_1 + \frac{(l_2 - l_1)(\frac{kN}{10} - cf)}{f}$, 6)Percentile $(P_k) = l_1 + \frac{(l_2 - l_1)(\frac{kN}{100} - cf)}{f}$

Measures Of Dispersion

7)Range = Large(L) - Small (s), Coefficient of Range = $\frac{L - S}{L + S}$

8) Quartile Deviation (Q.D) = $\frac{Q_3 - Q_1}{2}$, Coefficient of Q.D = $\frac{Q_3 - Q_1}{Q_3 + Q_1}$

9)Mean Deviation (M.D)

M.D from A = $\frac{\sum |x - A|}{n}$ (ungrouped frequency) , $\frac{\sum f\ |x - A|}{N}$ (grouped frequency)

Coefficient of M.D from A = $\frac{M.D\ from\ A}{A}$

Put A = Mean, to get M.D from Mean

A = Median, to get M.D from Median

A = Mode, to get M.D from Mode

10)Standard Deviation.

$\sigma_x = \frac{\sum (x - \bar{x})^2}{n}$ (Ungrouped), $\sigma_x = \frac{\sum f(x - \bar{x})^2}{N}$ (Grouped)

OR $\sigma_x = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$ (Ungrouped) , $\sigma_x = \sqrt{\frac{\sum fx^2}{N} - \bar{x}^2}$ (Grouped)

11)Coefficient of variation (C.V) = $\frac{standard\ deviation}{mean}\ x\ 100$

12)Combined Standard deviation (σ) = $\sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$,

Where $d_1 = \bar{x}_1 - \bar{x}$, $d_2 = \bar{x}_2 - \bar{x}$, $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

Unit 4 Probability

1)Experiment or trial : Any action, whose result is uncertain, not pre-decided . The result of an experiment is called Outcome.

Eg. Tossing a coin, throwing a dice

2) Sample Space: The set of all possible outcomes of an experiment .Denoted by S.

Eg. Tossing a coin S= {H,T} throwing a dice S = {1,2,3,4,5,6}

3) Event: Any sub set of a sample space .

Eg. {1,3,5}, {1,6}

4) Union of Events: A ∪ B is the event that either A or B or both take place.

5) Intersection of Events: A ∩ B is the event that both A and B take place.

6) Mutually Exclusive Events: A ∩ B is a null set(A ∩ B = ϕ)

7) Exhaustive Events: A ∪ B = Sample space (S)

8) Complementary Events: Complement of A is called A', such that A + A' = S

9) $P(E) = \frac{No.of\ outcomes\ favourable\ to\ E}{No.of\ possible\ outcomes} = \frac{n(e)}{n(S)}$, $0 \le P(E) \le 1$

10) P(E') = 1 – P(E)

11) Addition Theorem: P(A ∪ B)= P(A) + P(B) – P(A ∩ B)

12) If A and B are independent events then : P(A ∪ B)= P(A) + P(B)

13) If A and B are independent events then : P(A ∩ B)= P(A) . P(B)

14) **Expected Value** : $E(X) = \sum x.P(x)$,

$Variance: V(X) = E(X^2) - [E(X)]^2$, where $E(X^2) = \sum x^2.P(x)$

UNIT 5 DECISION THEORY

Events or States of Nature: These are the various occurrences which are outside decision maker's control.

$(Denote\ it\ by\ \ E_1, E_2, E_3 \ldots.\ or\ S_1, S_2, S_3 \ldots)$

Courses of action or Acts: These are the action or strategies which decision maker has control. (Denote it by $A_1, A_2, A_3 \ldots \ldots)$

1) Decision making under uncertainty

i) Maximax (optimistic) criterion

ii) Maximin (pessimistic) criterion

iii) Laplace criterion

iv) Minimax Regret Criterion

Events	Acts			
	A_1	A_2	A_3	A_4
E_1	30	80	90	100
E_2	70	30	50	80
E_3	120	10	40	20
Maximum	120	80	90	100
Minimum	30	10	40	20
Average	73.33	40	60	66.67

i) Maximax = Max(120,80,90,100) = 120. Select A_1

ii) Maximin = Max(30, 10, 40, 20)= 40.   Select A_3

iii) Laplace = Maxi (Average) = Maxi (73.33,40,60,66.67) = 73.33. Select A_1

iv) Minimax Regret Criterion: Same Question

Regret table:

Events	Acts			
	A_1	A_2	A_3	A_4
E_1	100-30=**70**	100-80=**20**	100-90=**10**	100-100=**0**
E_2	80-70=**10**	80-30=**50**	80-50=**30**	80-80=**0**
E_3	120-20=**0**	120-10=**110**	120-40=**80**	120-20=**100**
Maximum	70	110	80	100

Minimax = Min(70,110,80,100) = 70.   Select A_1

2) Decision making Under Risk

i) Expected Monetary Value (E.M.V)

Events	Acts			
	A_1	A_2	A_3	Probability
E_1	30	80	90	0.4
E_2	70	30	50	0.2
E_3	120	10	40	0.4

E.M.V of A_1 = (30x0.4) + (70x0.2) + (120x0.4) = 12+14+48 = 74

E.M.V of A_2 = (80x0.4) + (30x0.2) + (10x0.4) = 32 + 6 + 4 = 42

E.M.V of A_3 = (90x0.4) + (50x0.2) + (40x0.4) = 36+10+16 = 62

E.M.V of A_1 (74) is highest there for select A_1

ii) Expected Opportunity Loss (E.O.L): Same question

Make the regret table

Events	Acts			
	A_1	A_2	A_3	Probability
E_1	90-30= **60**	90-80= **10**	90-90= **0**	0.4
E_2	70-70= **0**	70-30= **40**	70-50= **20**	0.2
E_3	120-120= **0**	120-10=**110**	120-40= **80**	0.4

E.O.L of A_1 = (60x0.4) + (0x0.2) + (0x0.4)= 24+0+0=24

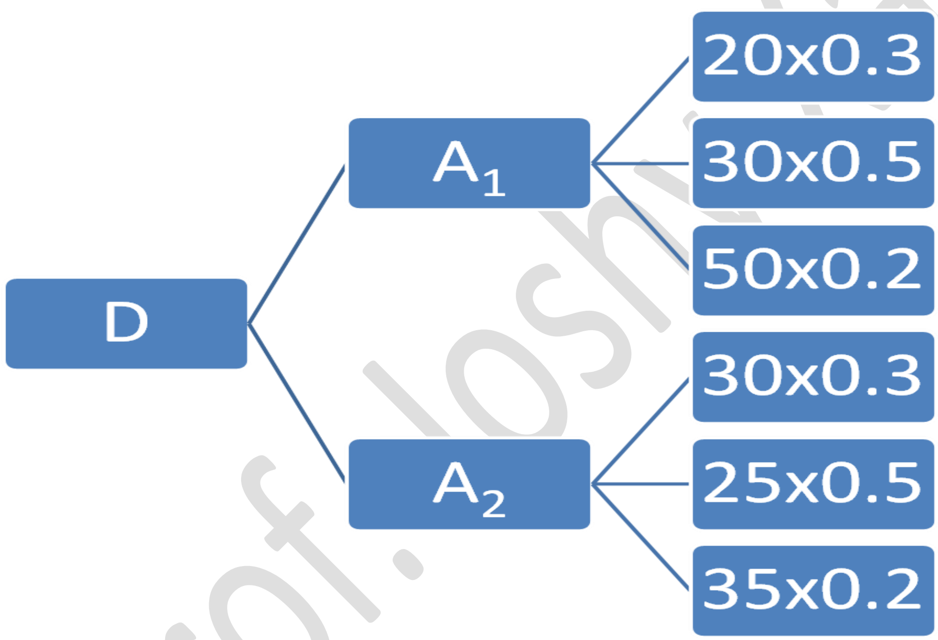
E.O.L of A_2 = (10x0.4) +(40x0.2) +(110x0.4)= 4+8+44=56

E.O.L of A_3 = (0x0.4) +(20x0.2) +(80x0.4)= 0+4+32=36

E.O.L of A_1 (24) is minimum there for select A_1

iii) Decision Tree

Events	Acts		
	A_1	A_2	Probability
E_1	20	30	0.3
E_2	30	25	0.5
E_3	50	35	0.2



E.M.V of A_1 = (20x0.3) + (30x0.5) + (50x0.2) = 6+15+10 = 31

E.M.V of A_2 = (30x0.3) + (25x0.5) + (35x0.2) = 9 + 12.5+ 7 = 28.5

E.M.V of A_1 (31) is highest there for select A_1